Multi Dimensional Multi Objective Transportation Problem by Goal Programming

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Abstract— Goal programming problem is similar to the linear programming problems. It extends the linear programming formulation to contain mathematical programming with multiple objectives. Goal programming techniques of multi-objective transportation problem (MOTP) have focused upon single dimension i.e. time, cost etc. In real life situation, all the transportation problems are not single dimensional but multi dimensional. So, we have to go for multi-dimensional MOTP. In this paper we solved multi-dimensional MOTP using goal programming problem and also found out optimum value of cost and time by R software.

Index Terms— Goal programming, Transportation Problem, Multi-objective transportation problem, Multi-dimensions.

1 Introduction

Goal programming (GP) techniques are used to solve a multi-objective optimization problem that balances trade-off in conflicting objective i.e. GP techniques helps in attaining the satisfactory level of all objectives. The method of formulating a mathematical model of GP is same as that of Linear Programming problem. There is little difference in linear programming and goal programming problems. The LP has two major limitations from its application point of view- single objective function and same unit of measurement of various resources. Whereas in GP model allows ordinal ranking of goal in terms of their contribution or importance to the organization. It may not be possible to obtain information about the value of a goal or sub-goal; therefore their upper and lower limits are determined. The desired goals are assigned priorities and then these priorities are ranked in an ordinal sequence. The concept of GP was introduced by Channes and Cooper (1961). They suggested a method for solving an infeasible LP problem arising from various goals. An importance of GP is that the goals are satisfied in ordinal sequence means the solution of the GP problem involves achieving some higher order goals first, before the lower order goals.

The GP can be solved by using two algorithms based on representing the multiple goals by a single objective function like weights methods and preemptive methods. The Weights methods consists the single objective function is the weighted sum of the functions representing the goals of the problem. The preemptive method start by prioritizing goals i.e. the initial focus should be on achieving first-priority goal. The other goals further divides into second-priority goal, third-priority goal, and so on.

The basic approach of goal programming is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals. There are three possible types of goals. A lower, one-sided goal sets a lower limit that we do not want to fall under (but exceeding the limit is fine); an upper, one-sided goal sets an upper limit that we do not want to exceed (but falling under the limit is fine) and a two-sided goal sets a specific target that we do not want to miss on either side.

2 General Mathematical Model

Goal Programming

The general goal linear programming model as follows:

$$Minimize: Z = \sum_{i=1}^{m} w_i P_i (d_i^+ + d_i^-) \qquad (1)$$

Subject to linear constraints

$$\sum_{j=1}^{n} a_{ij}\ x_j - d_i^+ + d_i^- = b_i; \qquad (2)$$
$$for\ i = 1,2,3, \dots \dots, m$$

And $\quad x_j, d_i^-, d_i^+ \geq 0, \quad for\ all\ i\ and\ j$

$$d_i^- \times d_i^+ = 0$$

Where Z is the sum of the deviations from all desired goals. The w_i are non- negative constant representing the relative weight to be assigned to the deviational variables d_i^+, d_i^-, within a priority level. The P_i is the priority level assigned

to each relevant goal in rank order. The a_{ij} are constant attached to each decision variables and b_i are the right hand side values (i.e. goals) of each constraint.

3 Multi-objective Transportation Problems

In real life situations, the transportation problem (TP) usually involves multiple, conflicting and incommensurate objective functions. This type of problems is called multi-objective transportation problem (MOTP). The mathematical model of MOTP can be stated as follows:

$$\min Z^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij} \ ,$$

Subject to

$$\sum_{j=i}^{n} x_{ij} = a_{i,} \qquad i=1,2,\ldots\ldots m$$

,

$$\sum_{i=1}^{m} x_{ij} = b_j \ ; \qquad j=1,2,3,\ldots..n$$

$$x_{ij} \geq 0, \qquad i = 1,2,\ldots\ldots,m; \ j = 1,2,\ldots\ldots,n; \ k = 1,2,\ldots\ldots,k$$

Where $Z^k(x) = \{Z^1(x), Z^2(x), \ldots\ldots\ldots, Z^k(x)\}$ is a vector of k objective functions, the superscript on both $Z^k(x)$ and C_{ij}^k is used to identify the number of objective functions (k = 1,2,…..,k), and m and n are the number of sources and destinations respectively. Without the loss of generality, it will be assumed in the paper that

$$a_i > 0 \quad \forall\, i, \qquad b_j > 0 \quad \forall\, j, \qquad C_{ij}^k \geq 0 \ \forall\, i,j$$

And

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

4 Model Formulation

Now, for this model is illustrated by one example. Consider the following 4X4 time and cost transportation problem. The upper left corner in each cell gives the time of transportation on the corresponding route and lower right corner in each cell gives the unit transportation cost per unit on that route. During the planning period, dealer is unable to meet whole demand of customer. However, dealer has to fulfill demand of all customers within time i.e. reliability of demand of the each customer from his suppliers has to be met and to minimize the total transportation cost and total transportation time. Transportation time and cost from i[th] source to j[th] destination given in the following table.

Destination→ Sources ↓	A	B	C	D	Supply
S_1	24 (x_{11}) 14	29 (x_{12}) 21	18 (x_{13}) 18	23 (x_{14}) 13	21
S_2	33 (x_{21}) 24	20 (x_{22}) 13	29 (x_{23}) 21	32 (x_{24}) 23	24
S_3	21 (x_{31}) 12	42 (x_{32}) 30	12 (x_{33}) 9	20 (x_{34}) 11	18
S_4	25 (x_{41}) 13	30 (x_{42}) 22	19 (x_{43}) 19	24 (x_{44}) 14	30
Demand	13	22	26	32	93

Let P be the priority level of goal. Here we assume that P_1, P_2 and P_3 are the priority levels of the goals. X_{ij} be the amount to be transported from i[th] supplier to j[th] destination.

P_1 : Demand of all customers must be satisfied i.e. reliability in transportation.

P_2 : Minimize the total transportation cost.

P_3 : Minimize the total transportation cost.

Let;

$d_i^+ = over\ achievement\ of\ the\ goals\ or\ constaints\ in\ the\ i^{th}\ equation$

$d_i^- = under\ achievement\ of\ the\ goals\ or\ constaints\ in\ the\ i^{th}\ equation$

MOTP with multi dimensional goal programming model, first we have to formulate the model constraints on the basis of our goals.

P_1: Dealer has determined that demand of all customers must be satisfied within time i.e. reliability of the each customer from his suppliers or dealer is met.

$$24x_{11} + 33x_{21} + 21x_{31} + 25x_{41} + d_1^- - d_1^+ = 1 \qquad (6)$$
$$29x_{12} + 20x_{22} + 42x_{32} + 30x_{42} + d_2^- - d_2^+ = 22 \qquad (7)$$
$$18x_{13} + 29x_{23} + 12x_{33} + 19x_{43} + d_3^- - d_3^+ = 26 \qquad (8)$$

$$23x_{14} + 32x_{24} + 20x_{34} + 24x_{44} + d_4^- - d_4^+ = 32 \qquad (9)$$

P_2: Minimize the total transportation cost.

$$\sum x_{ij}\, c_{ij} + d_5^- - d_5^+ = 0;\ for\ i,j$$

i.e.

$$14x_{11} + 21x_{12} + 18x_{13} + 13x_{14} + 24x_{21} + 13x_{22} + 21x_{23} + 23x_{24} + 12x_{31} + 30x_{32} + 9x_{33} + 11x_{34} + 13x_{41} + 22x_{42} + 19x_{43} + 14x_{44} + d_5^- - d_5^+ = 0 \qquad (10)$$

P_3: Minimize the total transportation time.

$$\sum x_{ij}\, t_{ij} + d_6^- - d_6^+ = 0;\ for\ i,j$$

i.e.

$$24x_{11} + 29x_{12} + 18x_{13} + 23x_{14} + 33x_{21} + 20x_{22} + 29x_{23} + 32x_{24} + 21x_{31} + 42x_{32} + 12x_{33} + 20x_{34} + 25x_{41} + 30x_{42} + 19x_{43} + 24x_{44} + d_6^- - d_6^+ = 0 \qquad (11)$$

Hence, MOTP with multi dimensional goal programming model given as follows,

Minimize

$$Z = P_1(d_1^- + d_2^- + d_3^- + d_4^-) + P_2 d_5^+ + P_3 d_6^+$$

Subject to;

$$24x_{11} + 33x_{21} + 21x_{31} + 25x_{41} + d_1^- - d_1^+ = 13$$
$$29x_{12} + 20x_{22} + 42x_{32} + 30x_{42} + d_2^- - d_2^+ = 22$$
$$18x_{13} + 29x_{23} + 12x_{33} + 19x_{43} + d_3^- - d_3^+ = 26$$
$$23x_{14} + 32x_{24} + 20x_{34} + 24x_{44} + d_4^- - d_4^+ = 32$$
$$14x_{11} + 21x_{12} + 18x_{13} + 13x_{14} + 24x_{21} + 13x_{22} + 21x_{23} + 23x_{24} + 12x_{31} + 30x_{32} + 9x_{33} + 11x_{34} + 13x_{41} + 22x_{42} + 19x_{43} + 14x_{44} + d_5^- - d_5^+ = 0$$
$$24x_{11} + 29x_{12} + 18x_{13} + 23x_{14} + 33x_{21} + 20x_{22} + 29x_{23} + 32x_{24} + 21x_{31} + 42x_{32} + 12x_{33} + 20x_{34} + 25x_{41} + 30x_{42} + 19x_{43} + 24x_{44} + d_6^- - d_6^+ = 0$$
$$x_{ij}, c_{ij}, t_{ij}, d_i^- d_i^+ \geq 0$$

Z= {13, 22, 26, 0, 17, 32}, these are the values of d1M, d2M, d3M, d4M, d5M and d6M. From this result, we have to say that in priority level 1 or goal 1 demand of customer A, B and C cannot be achieved because value of d1M is 13, d2P is 22 and C is 26 i.e. in priority 1 or goal 1 only customer D has satisfy his whole demand. So then we say that supplier has cannot be supply to his customer in within time i.e. he is not reliable. Hence goal 1 cannot be achieved. Priority 2 and 3 are related to time and cost, these are also cannot be achieved because in transportation cost and time never zero.

Optimum value of time and cost by R software is given by 1896 and 1200 respectively.

5 Conclusions:

In real life situation, all the transportation problems are not single dimension but sometimes they may be with the multi-dimensions. So, we have to go for multi-dimensions MOTP. The present paper introduces a new technique of solving multi-dimensions MOTP in a single formulation of transportation problem. The method has been illustrated with suitable numerical example using LINDO 14.0 software. The goals achieved for this problem is only customer D. The present work is most useful to those researchers working on multi-dimensions MOTP by using goal programming problem or any other techniques.

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